Modeling of Aircraft Position Errors with Independent Surveillance

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In order to reduce present air traffic separation standards, a means of quantitatively measuring the safety level of a particular air route structure must be established. The most important factor in determining route safety is the distribution of aircraft position errors about their intended tracks. This paper presents a modeling technique which can compute the probability density function of position errors as the aircraft proceed along the route. The technique takes into account not only the time dependence, but also all the factors influencing an aircraft's position errors, e.g., surveillance and navigation errors, surveillance fix rate, and air traffic control procedures.

I. Introduction

CURRENT projections for traffic increase over most domestic and oceanic air routes, especially in the North Atlantic Region, indicate that, with present separation standards† between routes, many carriers will experience heavy economic penalties due to flying nonoptimum routes. One of the primary means of providing for the reduction of the separation standards, thus minimizing economic penalties, is through the use of an independent surveillance system, e.g., satellite surveillance. These systems would operate independently of aircraft onboard navigation systems and would provide position updating for those aircraft with large position errors. However, before any of the separation standards are reduced, an adequate safety level for the projected route structure must be guaranteed.

In this paper, a method is presented for modeling the behavior of aircraft, including the influence of an independent surveillance system, in order to obtain a time-dependent description of the distribution of aircraft position errors. This position error distribution can be used to calculate the probability of collision between aircraft on adjacent routes, and the associated safety level. The model in this paper is considered an improvement over existing position error models[1-5] because it considers both the factors that cause midair collisions—such as navigation system errors, surveillance system positioning errors—and the factors that help prevent them—route structure, surveillance fix rate, air traffic control procedures.

Only the problem of modeling aircraft position errors in the lateral dimension and with a strategic air traffic control strategy is considered in this paper. Reference 6 considers the longitudinal dimension and tactical control scheme, as well as the case discussed here.

II. Calculation of Safety for Given Route Structure

The problem of quantitatively assessing the safety of the North Atlantic route structure as a function of such factors as the probability of collision between two aircraft on adjacent tracks, airline scheduling, and aircraft characteristics is presently being considered by the North Atlantic Systems Planning Group (NATSPG),[7-9] made up of representatives of six North Atlantic countries. This group has decided to adopt the collision risk equation formulated by P. G. Reich[9-11] as the means of relating the route structure to safety, which has been defined as the number of accidents in 10 million flying hours.

The principal feature of the Reich equation is the inclusion of an exposure term, which is intended to weight the probability of collision between two aircraft by the probability that they were in the given orientation when separation was lost. This exposure term is extremely important, and has very often been neglected in safety analyses. The formulation of the Reich equation, however, includes two very basic assumptions. a) Aircraft position errors in each dimension are independent. b) Position errors for aircraft on adjacent tracks occur independently. A discussion of the consequences of these assumptions is beyond the scope of this paper and the reader is referred to Ref. 6.

The most important inputs to the Reich equation are the probabilities of collision between two aircraft, on adjacent tracks, due to a loss of separation in each of the three dimensions. This paper is concerned with the lateral dimension, and by assumption b, only the position errors of aircraft along a single track need be considered. The analysis proceeds by first computing the probability of collision for two aircraft which are side-by-side and then scaling this by the exposure factor.

III. Calculation of Probability of Lateral Collision

The probability of lateral collision (overlap) between aircraft on adjacent lanes is computed from the lateral position error probability density functions for each of the aircraft about its respective intended trajectory. Most important of all is the probability of an aircraft having a lateral position error in excess of one-half the lateral separation standard. Clearly, if each aircraft had zero probability of being beyond the half-standard, the probability of collision between the two aircraft would be zero. The portion of the position error density function for large position errors is referred to as the "tails."

An important characteristic of the collision probability between aircraft on adjacent lanes, especially with independent surveillance, is that it is definitely time-dependent. This feature has often been neglected in previous approaches to aircraft position error modeling, and as a consequence, worst case errors had to be used. This leads to a very pessimistic assessment of the safety level for the given route structure.

As stated, the quantity that specifies the probability of collision, and therefore the safety level of a given route structure, is the probability density function of an aircraft's position errors about its intended track. The next section indicates the modeling technique for calculating this function.

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† 120 naut miles lateral, 2000 ft vertical and 15 min longitudinal in the North Atlantic region.

IV. Calculating the Probability Density Function of Lateral Position Errors

There are essentially three separate elements which affect an aircraft's lateral position. They are: 1) the navigation system errors, causing the aircraft to drift away from its intended trajectory, 2) the surveillance positioning errors, which prevent an aircraft from accurately returning to its intended track (even in the absence of navigation errors), after it has been given an "alarm" for appearing to have a large position error, and 3) the ATC procedures that determine the type of track structure and the conditions under which surveillance "alarms" will be given.

Any modeling procedure for calculating the probability of an aircraft having a certain error at a given time must, therefore, include these three elements and, most important, a means of modeling their effects on the aircraft.

Of the three elements, the ATC procedures is probably the most unfamiliar. The operation of a strategic ATC/Surveillance System incorporates some form of a threshold or boundary, which the aircraft must not cross. This threshold can be either a position threshold (some intermediate boundary inside the half-standard) or a time threshold (time to go before crossing the half-standard). The model in this paper employs a position threshold and the operation of the ATC/Surveillance System is assumed to proceed as follows.

Aircraft enter the track structure with some specified distribution of cross-track position errors with a standard deviation of σ_1 naut miles. As the aircraft proceed, the navigation system errors cause an additional random cross-track position drift with a standard deviation of σ_n knots. A surveillance system monitors the position of each aircraft every T_0 minutes. The surveillance system positioning accuracy is assumed to be gaussian distributed with a standard deviation of σ_S naut miles. As long as the aircraft appear to the surveillance system to be within the position, or alarm threshold, r_0 naut miles, they are allowed to proceed unhindered. If an aircraft appears to have a cross-track error of more than r_0 naut miles, its "present position" is updated with the surveillance system's estimate of its position and it is issued a return heading, θ_h, and velocity V_h.

These are the factors which will determine the statistics of an aircraft's cross-track position error. Several others, including the human blunder factor, have been neglected. Although all of the random factors, such as navigation drift rate and surveillance accuracy, may have gaussian statistics, the over-all density function of cross-track position errors will not be gaussian due to the introduction of the alarm threshold. All aircraft appearing to be beyond the alarm threshold are sent back to the intended track at every surveillance fix time, which has the effect of "folding" the tails of the position error density function onto the body (that portion between alarm thresholds). This is a nonlinear operation, resulting in a nongaussian distribution of errors.

The procedure by which the model generates the density function of cross-track position errors is as follows.

1) With the density function of position errors specified at the first surveillance fix time, the position error probability density function is evaluated at the second surveillance fix time at N (nominally 20) values of possible (positive‡) errors.§ (If the density function is desired at any intermediate time, the same procedure is followed.)

2) An interpolation routine (for the N points) is used to describe the position error probability density function at the second surveillance fix time.

3) The resulting density function is used (together with the initial density function) to generate the probability density function of lateral position errors at the N points for any time up to and including the third surveillance fix time.

4) The interpolation routine is used again, and the entire process repeated. In general, as many past surveillance fix time position error density functions will be required to describe the density function during the next interval as the maximum number of surveillance fix intervals it takes an aircraft to return to the intended track after a surveillance alarm.

As can be seen from these descriptions, the calculation of the probability of an aircraft having a specific position error at a given time, as a function of the ensemble of past errors, is the keystone of this model. The computation of this probability as an element of the total lateral position error density function is described in detail in Appendix A for the case of aircraft returning to the intended track in a maximum of two fix intervals. The total computation requires the numerical evaluation of nine integrals.

Every possible trajectory is included in one of the integral terms. Each integral, therefore, represents a certain category of trajectory an aircraft can fly and at the desired time have a specified lateral position error. What makes this representation possible is that the form of the integral relationships describing the probability of being at a certain point at a desired time (as a function of the position error density functions at the previous fix times) is independent of the particular fix interval being considered. The only factors that change are the parameters describing the navigation and surveillance accuracy (as a function of time). The form of the expressions that incorporate these accuracies is invariant.

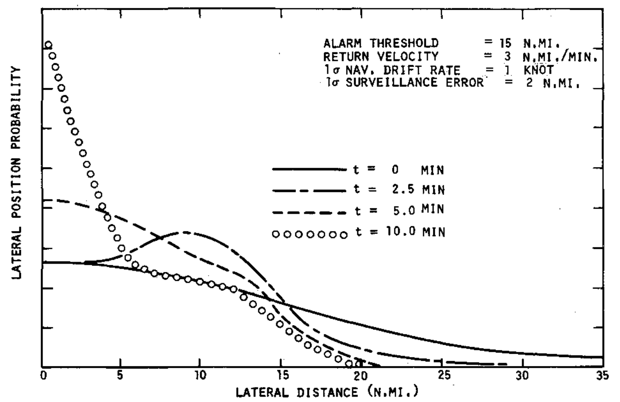

Fig. 1 Change in the distribution of cross-track errors between two surveillance fixes.

V. Numerical Example

Included in this section is a numerical example which illustrates the operation of the model presented in Sec. IV and Appendix A. The variation in the lateral position error density function for several times between two surveillance fixes is described. The values assumed for the several model parameters are indicated as follows: r_0 = alarm threshold = 15 naut miles, σ_n = INS (navigation system) error drift rate = 1 knot, σ_s = surveillance system positioning accuracy = 2 naut miles, T_0 = surveillance fix interval = 10 min, and σ_I = standard deviation of initial distribution = 15 naut miles. In this example it is assumed that, initially, the aircraft lateral position errors are gaussian distributed with standard deviation σ_I. At time 0 the surveillance system monitors the aircraft position and issues return trajectories to those aircraft which appear to be beyond the alarm threshold. It is assumed that the return trajectories are all parallel and the return (cross-track) velocity is 180 knots. Using the model, the lateral position error density function is calculated at intervals of 2.5 min between 0 and 10 min. The results are given in Fig. 1. Note that only half the density function is given since it is symmetric about zero (assuming zero bias).

‡ Since the density function is symmetric, only positive values of position error need be considered.

§ The probability of a single position error is, strictly speaking, zero because of the continuous density function. Reference to the probability of the aircraft being at a certain point or having a specific position error is meant to imply a small neighborhood, dy, about the indicated position.

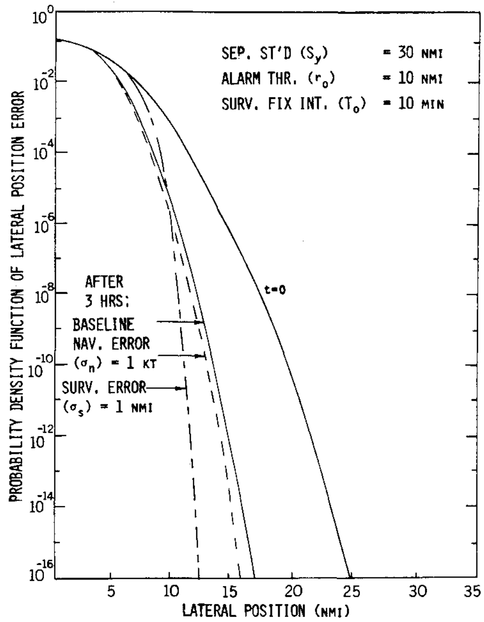

Fig. 2 Variation in position error probability density function.

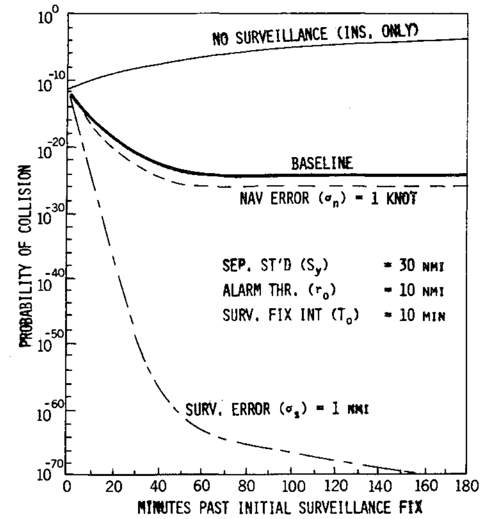

Fig. 3 Probability of collision as a function of time.

Potentially, every aircraft beyond 15 naut miles should receive a surveillance alarm. However, the surveillance accuracy is 2 naut miles which causes some aircraft with errors less than 15 naut miles to receive an alarm. At 2.5 min, there is definitely a wave of aircraft returning to the intended track (0 naut miles position error). These aircraft will be redistributed about the intended track with a gaussian distribution. The standard deviation of this redistribution will be approximately 2 naut miles since the position errors at the end of the return maneuvers are due mainly to the original surveillance positioning error. At later times (7.5 and 10 min), the position error density function develops a sharp peak at the center, while the tails continue to decrease. Since the duration of this example is only 10 min, the lateral drift resulting from the navigation system errors (with a 1σ value of $\frac{1}{6}$ naut miles) is insignificant.

VI. Sensitivity of Position Error Distribution and Probability of Collision to Parameter Variations

As indicated in the derivation of the position error model in Sec. IV, there are a number of parameters which influence the behavior of an aircraft. In the design and specification of any surveillance system, it is important to know which of these parameters has the most influence on the distribution of lateral position errors, the probability of collision and, ultimately, the route safety. In this section, the sensitivity of the lateral position error density function and the probability of collision is given with respect to variations in the following parameters: surveillance accuracy, navigation system error, and initial distribution of position errors. From previous analysis it was found that these were among the more important parameters, and, moreover, they are the ones most likely to be influenced by technological changes. It is important to realize that these are only a few examples of the type of sensitivity analyses that can be performed with the model presented in this paper. In fact, the value for each of the parameters can be varied. Moreover, since the algorithm is built in modular form, the entire navigation and surveillance system position error density function can be changed.

To perform the sensitivity study given here, a baseline set of parameter values was first specified, and then several individual parameters were varied. Of course, this only gives an indication of the effect of single parameter variations, but these examples are meant principally to illustrate the operation of the model.

The baseline set of parameter values is: surveillance fix interval —(T_0) 10 min, alarm threshold—(r_0) 10 naut miles, lateral separation standard—(S_y) 30 naut miles, standard deviation of navigation system drift error—(σ_n) 2 knots, standard deviation of surveillance positioning error—(σ_S) 2 naut miles, standard deviation of initial aircraft position error distribution—(σ_I) 3 naut miles, return heading angle¶—(θ_h) 20°, return velocity—(V_h) 180 knots.

Figure 2 gives the sensitivity of the tails of the position error density function to changes in the navigation error and surveillance error, and as a function of time. Since the initial distribution of aircraft is assumed the same for all conditions (except when it is the object of the sensitivity analysis), the (one-sided) probability density function for $t = 0$ is constant. Keeping in mind that the alarm threshold is at 10 naut miles and the half-standard is at 15 naut miles, it is clear that a factor of two decrease in the surveillance error had a much more dramatic effect on the tails than the same change in the navigation error. This would imply that the probability of collision is going to be much more sensitive to the surveillance error than the navigation error.

This is verified in Fig. 3, which gives the probability of collision as a function of time for changes in the surveillance and navigation errors. Also shown is the probability of collision for the case when no surveillance is assumed. The position errors, in this latter case, increase due to the navigation system drift error.

Although the probability of collision curves reach steady-state (except when $\sigma_S = 1$ naut mile), the extreme sensitivity of the probability of collision to the value of the surveillance error indicates that if this parameter varied as a function of position, there would not be any steady-state collision probability. Moreover, even this relatively simple sensitivity study indicates the value of including time dependence in the position error model, since the baseline collision probability varies over 10 orders of magnitude before reaching steady state.

VII. Conclusions

A technique for computing the probability density function for aircraft lateral position errors as a function of time and including the effect of independent surveillance has been presented. Such a technique is the basis for the evaluation of the safety level of a parallel route structure using the probability of collision as the safety measure. Such an approach has already been accepted for the North Atlantic region. It is shown that the time dependence of the collision probability is very important and that for a

¶ Aircraft are assumed to return on parallel, straight-line trajectories.

strategic control strategy, the surveillance system positioning accuracy is the dominant ATC system parameter.

Appendix: Development of Aircraft Position Error Model for the Case of Aircraft Returning to the Intended Track within Two Fixes

In describing the development of this modeling technique, the attempt is made here to use general descriptions of the navigation and surveillance system error probability density functions. In addition, all the assumptions made in deriving the model, including those necessary for expanding the model to include more fix intervals, are detailed.

Definitions

The following variables and functions will be used in developing the model. T_0 = surveillance fix interval. r_0 = alarm threshold (lateral dimension). S_y = separation standard (lateral dimension). $f_{e_y(\Delta, nT_0)}(s)$ = probability density function of navigation error e_y (perpendicular to intended track**) between time nT_0; $n = 1, 2, 3 \ldots$ and $nT_0 + \Delta, 0 < \Delta \leqq T_0$. $f_{\varepsilon(nT_0)}(s)$ = probability density function of surveillance positioning error ε at time nT_0. $P_A(r, nT_0)$ = probability that an aircraft receives a surveillance alarm at time nT_0 if it is r naut miles from the intended track; $0 \leqq P_A(r, nT_0) \leqq 1$. $D(r, \Delta)$ = a real valued (positive) function which indicates the distance traveled towards the intended track Δ min after the aircraft receives a surveillance alarm and has started on a return trajectory. At the time of the alarm, it appeared (to the surveillance system) to be r naut miles from the intended track. Figure 4 shows the physical interpretation of $D(r, \Delta)$ for a typical return trajectory. If all aircraft return at a constant velocity V_y (neglecting finite radius of turn) dependent only on their observed position, then $D(r, \Delta) = V_y(r) \cdot \Delta$ (where V_y is a function only of r).

In order to keep track of aircraft which have return trajectories lasting longer than one surveillance fix interval, it is necessary to use the fact that if an aircraft's intended direction of flight is parallel to the desired track, it is not on a return trajectory. For this purpose, the following additional terms are defined. $Y_1(t, y)$ = probability density function of lateral position errors at time t for those aircraft with nominal direction of flight parallel to the intended track. $Y_2(t, y)$ = probability density function of lateral position errors at time t for those aircraft not having a nominal direction of flight parallel to the intended track. This implies that these aircraft are on a return trajectory after having been issued a surveillance alarm. $Y(t, y) \triangleq Y_1(t, y) + Y_2(t, y)$.

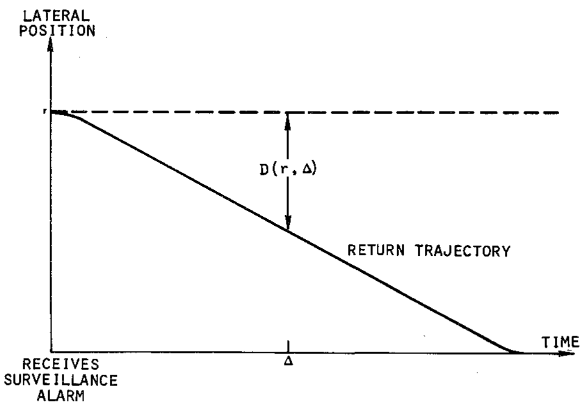

Fig. 4 Definition of $D(r, \Delta)$—distance traveled in Δ minutes towards intended track.

** The random navigation error source is assumed to change value only at surveillance fix times. The calculation of this lateral position error density function may include a geometric factor which is a function of the direction of flight relative to the desired trajectory.

Categorizing Surveillance Errors

Whether or not an aircraft is on a return trajectory at any given time depends on its observed position at the previous one or two surveillance fix times. It is the observed position which is important since it determines whether or not a surveillance alarm is to be given and what the return trajectory will be.

An aircraft's observed position at a surveillance fix time is a function of its actual position, r, and the surveillance positioning error, ε. For each r, and any time interval Δ since the last surveillance fix, the type of trajectory an aircraft is flying (e.g., whether on a return trajectory or not, whether it is to return in one or two fix intervals) can be determined by noting the relation of the surveillance errors at the last one or two surveillance fix times to six critical surveillance errors (which are a function of r, Δ and T_0). For the case where the aircraft appears to be beyond $+r_0$, the following are defined.†† 1) $\varepsilon^{\Delta} = [\varepsilon \,|\, r+\varepsilon = D(r+\varepsilon, \Delta)] \equiv$ that surveillance error so that an aircraft appearing to be at lateral position $r+\varepsilon$ will return to the intended track in Δ min. 2) $\varepsilon^{T_0} = [\varepsilon \,|\, r+\varepsilon = D(r+\varepsilon, T_0)]$. 3) $\varepsilon^{T_0+\Delta} = [\varepsilon \,|\, r+\varepsilon = D(r+\varepsilon, T_0+\Delta)]$.

For the case where the aircraft appears to be beyond $-r_0$, the next three are defined as follows: 4) $\varepsilon_{\Delta} = [\varepsilon \,|\, r+\varepsilon = -D(r+\varepsilon, \Delta)] \equiv$ that surveillance error so that an aircraft appearing to be at lateral position $-(r+\varepsilon)$ will return to the intended track in Δ min. 5) $\varepsilon_{T_0} = [\varepsilon \,|\, r+\varepsilon = -D(r+\varepsilon, T_0)]$. 6) $\varepsilon_{T_0+\Delta} = [\varepsilon \,|\, r+\varepsilon = -D(r+\varepsilon, T_0+\Delta)]$.

For an aircraft at lateral position r, if the surveillance error at the previous fix time is less than ε^{Δ} (but greater than $r_0 - r$) or greater than ε_{Δ} (but less than $-r_0 - r$), its estimated trajectory will intersect the intended track within Δ minutes of the last surveillance fix. Therefore, for any time greater than Δ, such an aircraft will have its intended trajectory parallel to the intended track. The similar situation exists for ε_{T_0}, ε^{T_0} and $\varepsilon_{T_0+\Delta}$, $\varepsilon^{T_0+\Delta}$.

Figure 5 depicts how, for a particular r, the range of possible surveillance errors is divided into nine bands, with the boundaries being the six surveillance errors defined earlier and the minimum surveillance errors $r_0 - r$ and $-r_0 - r$. These latter two are bonafide limits because an aircraft must appear to be outside the interval $[-r_0, r_0]$ in order to receive an alarm. These nine bands will play a fundamental role in describing aircraft behavior.

If the assumption of returning to the intended track in two fixes is changed to three fixes, then four more integrals need to be added. This will consider the additional types of return trajectories taking up to three fix intervals to complete. Four additional terms would be defined ε^{2T_0}, $\varepsilon^{2T_0+\Delta}$, ε_{2T_0}, and $\varepsilon_{2T_0+\Delta}$.

Model Description and Assumptions

Assuming that Y_1 and Y_2 have already been determined for surveillance fix times nT_0 and $(n-1)T_0$, this section will describe the calculation of $Y_{1,2}(nT_0+\Delta, y)$ for $0 < \Delta \leqq T_0$ and any lateral position, y. Note that by setting $\Delta = T_0$, $Y((n+1)T_0, y)$ can be found, enabling the calculations to proceed another step. The equations for $Y_{1,2}(nT_0+\Delta, y)$ contain nine integral expressions, each one corresponding to one of the surveillance system error ranges. Each range determines a different type of trajectory that an aircraft can follow and arrive at a particular place at time $nT_0 + \Delta$.

There are two basic assumptions that are made in order to simplify the description of the aircraft's lateral behavior. They are 1) The error source in the navigation system is allowed to change value only at surveillance fix times. This change will account for all random perturbations throughout the previous interval. The possibility of a change in error source is assumed,

†† In each case $0 < \Delta \leqq T_0$.

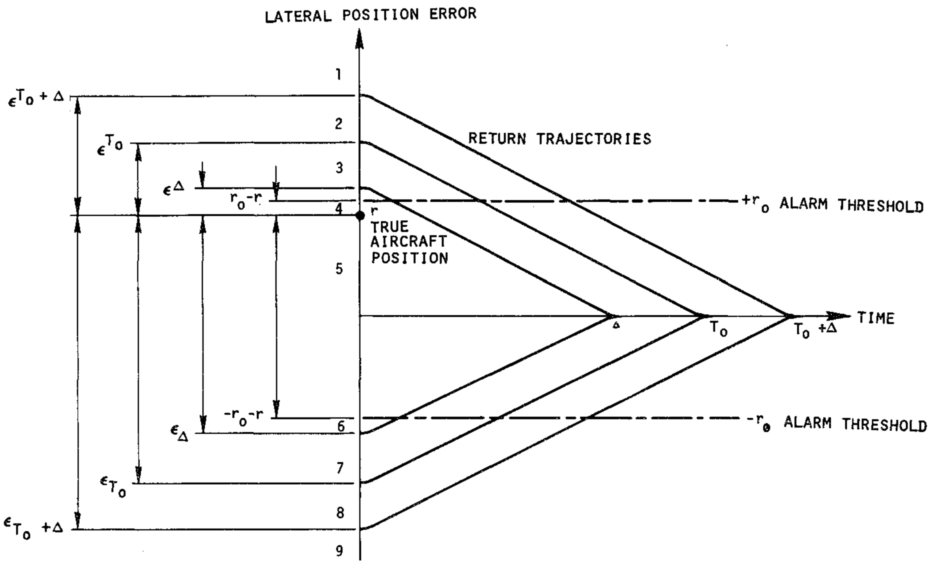

Fig. 5 Range of surveillance errors and effect on return trajectory.

whether a heading correction is or is not made. 2) For aircraft on return trajectories and receiving no additional surveillance alarms, the random navigation error is not allowed to change until the next surveillance fix time after the aircraft's assumed position has returned to the intended track. This second simplifying assumption is necessary in order to accommodate the condition that aircraft can take as long as two fixes to return to the intended track.

If an aircraft gets another surveillance alarm while still on its return trajectory, it will be issued an entirely new return trajectory command independent of the path it took to arrive at its current position. In such a case, the random navigation factor is allowed to change at the intermediate surveillance fix time. For example, if an aircraft receives an alarm at $(n-1)T_0$ and is issued a return trajectory which will return the aircraft to the intended track at time $nT_0+\Delta$ and the aircraft receives another alarm at nT_0, the random navigation error (e.g., drift rate naut miles/min) is assumed constant over the interval $[nT_0, (n+1)T_0]$.

Under the conditions imposed by the two modeling assumptions, the surveillance model for $Y(nT_0+\Delta, y)$ is given by

$$Y(nT_0+\Delta, y) = Y_1(nT_0+\Delta, y) + Y_2(nT_0+\Delta, y) \qquad \text{(A1)}$$

$$Y_1(nT_0+\Delta, y) = \int_{-\infty}^{\infty} \left(1-P_A(r, nT_0)\right) Y_1(nT_0, r) f_{e_y(\Delta, nT_0)}(y-r)\,dr \qquad \text{(A2)}$$

This first term gives the position error probability density function at time $nT_0+\Delta$ for aircraft that at time nT_0 had a heading parallel to the intended track and did not receive a surveillance alarm (see Fig. 6)

$$+\int_{-\infty}^{\infty} Y(nT_0, r) \int_{r_0-r}^{\max\{\varepsilon^{\Delta}, r_0-r\}} f_{e_y(\Delta, nT_0)}(y+s) f_{\varepsilon(nT_0)}(s)\,ds\,dr \qquad \text{(A3)}$$

This second term gives the position error probability density function at time $nT_0+\Delta$ for aircraft that appeared to the surveillance system to be beyond r_0 at nT_0, and have returned to the intended track by $nT_0+\Delta$ (see Fig. 7)

$$+\int_{-\infty}^{\infty} Y(nT_0, r) \int_{\min\{\varepsilon_{\Delta}, -r_0-r\}}^{-r_0-r} f_{e_y(\Delta, nT_0)}(y+s) f_{\varepsilon(nT_0)}(s)\,ds\,dr \qquad \text{(A4)}$$

This third term is the same as Eq. (A3), except the aircraft appeared to be beyond $-r_0$ at nT_0 (see Fig. 7)

$$+\int_{-\infty}^{\infty} Y((n-1)T_0, r) \int_{\max\{r_0-r, \varepsilon^{T_0}\}}^{\max\{r_0-r, \varepsilon^{T_0+\Delta}\}} f_{e_y(T_0+\Delta, (n-1)T_0)}(y+s) \times [1-P_A\{r-D(r+s, T_0)+e_y(T_0, (n-1)T_0)\}] f_{\varepsilon[(n-1)T_0]}(s)\,ds\,dr \qquad \text{(A5)}$$

This fourth term gives the position error probability density function at time $nT_0+\Delta$ for aircraft that appeared to be beyond r_0 at $(n-1)T_0$, were still on their return trajectory at nT_0 and did not receive an alarm at nT_0, and have returned to the intended track by $nT_0+\Delta$. The term $e_y[T_0, (n-1)T_0]$ is the

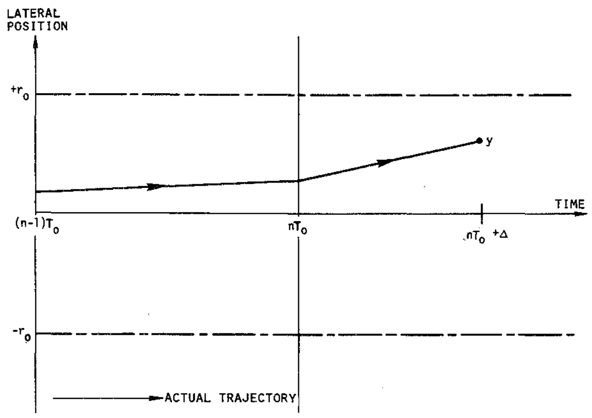

Fig. 6 Aircraft trajectory when no alarm is received at *n*th fix time.

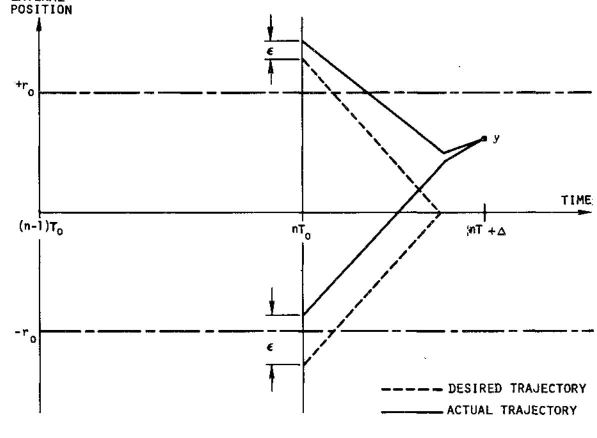

Fig. 7 Aircraft trajectory when alarm is received ay *n*th fix time and return maneuver is complete within Δ min.

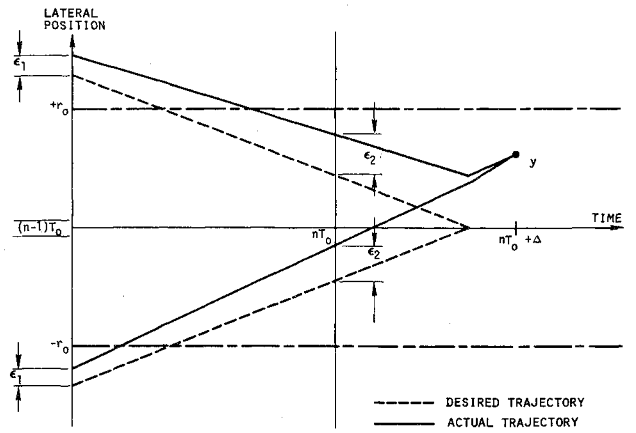

Fig. 8 Aircraft trajectory when alarm received at $(n-1)^{st}$ fix time and return maneuver is complete within $T_0+\Delta$ min.

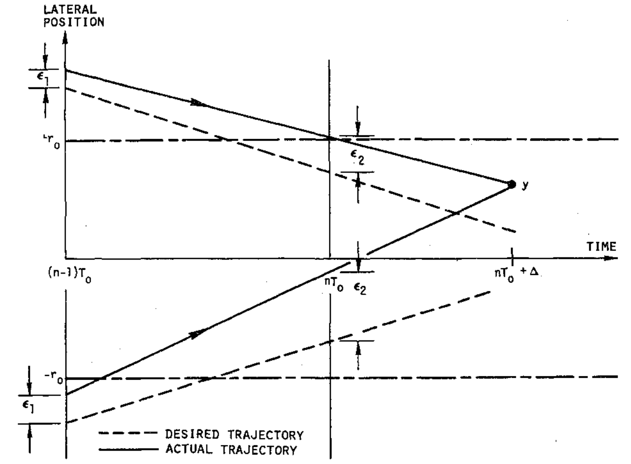

Fig. 10 Aircraft trajectory when alarm is received at $(n-1)^{st}$ fix time and return maneuver is not complete within $T_0+\Delta$ min.

navigation error at time nT_0 using the same value of the error source that existed at $(n-1)T_0$ (see Fig. 8)

$$+\int_{-\infty}^{\infty} Y[(n-1)T_0, r] \int_{\min\{-r_0-r,\varepsilon_{T_0+\Delta}\}}^{\min\{-r_0-r,\varepsilon_{T_0}\}} f_{e_y(T_0+\Delta,\,(n-1)T_0)}(y+s) \times$$
$$[1-P_A\{r+D(r+s,\,T_0)+e_y(T_0,(n-1)T_0)\}]\cdot f_{\varepsilon[(n-1)T_0]}(s)\,ds\,dr \tag{A6}$$

This fifth term is the same as Eq. (A5), except the aircraft appeared to be beyond $-r_0$ at $(n-1)T_0$ (see Fig. 8)

$$Y_2(nT_0+\Delta, y) = \int_{-\infty}^{\infty} Y(nT_0, r) \int_{\max\{\varepsilon^{\Delta}, r_0-r\}}^{\infty} f_{e_y(\Delta,\,nT_0)} \times$$
$$[y-r+D(r+s,\Delta)]f_{\varepsilon(nT_0)}(s)\,ds\,dr \tag{A7}$$

This sixth integral gives the position error probability density function at time $nT_0+\Delta$ for aircraft that appeared to be beyond r_0 at nT_0 and have not returned to the intended track by $nT_0+\Delta$ (see Fig. 9)

$$+\int_{-\infty}^{\infty} Y(nT_0, r) \int_{-\infty}^{\min\{\varepsilon_{\Delta}, -r_0-r\}} f_{e_y(\Delta,\,nT_0)}(y-r-D(r+s,\Delta)) \times$$
$$f_{\varepsilon(nT_0)}(s)\,ds\,dr \tag{A8}$$

This seventh integral is the same as Eq. (A7) except the aircraft appear to be beyond $-r_0$ at nT_0 (see Fig. 9)

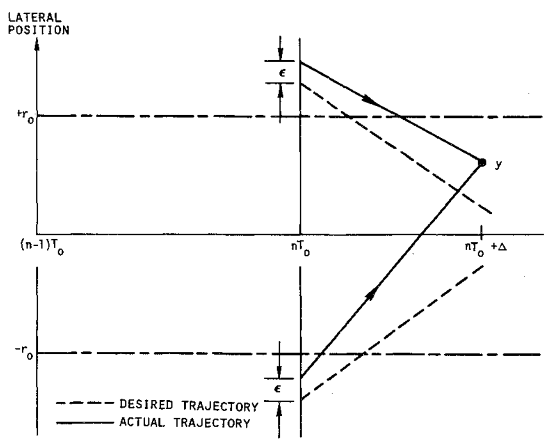

Fig. 9 Aircraft trajectory when alarm is received at nth fix time and return maneuver is not complete within Δ min.

$$+\int_{-\infty}^{\infty} Y((n-1)T_0, r) \int_{\max\{r_0-r,\varepsilon^{T_0+\Delta}\}}^{\infty} f_{e_y(T_0+\Delta,\,(n-1)T_0)} \times$$
$$[y-r+D(r+s,\,T_0+\Delta)](1-P_A\{r-D(r+s,\,T_0)+$$
$$e_y[T_0,(n-1)T_0]\})\cdot f_{\varepsilon[(n-1)T_0]}(s)\,ds\,dr \tag{A9}$$

This eighth integral gives the position error probability density function at time $nT_0+\Delta$ for aircraft that appeared to be beyond r_0 at $(n-1)T_0$, were still on their return trajectory at nT_0 and did not receive an alarm, and have not returned to the intended track by $nT_0+\Delta$ (see Fig. 10)

$$+\int_{-\infty}^{\infty} Y[(n-1)T_0, r] \int_{-\infty}^{\min\{-r_0-r,\varepsilon_{T_0+\Delta}\}} f_{e_y[T_0+\Delta,\,(n-1)T_0]} \times$$
$$[y-r-D(r+s,\,T_0+\Delta)](1-P_A\{r+D(r+s,\,T_0)+$$
$$e_y[T_0,(n-1)T_0]\})\cdot f_{\varepsilon[(n-1)T_0]}(s)\,ds\,dr \tag{A10}$$

This ninth integral is the same as Eq. (A9) except the aircraft appeared to be beyond $-r_0$ at $(n-1)T_0$ (see Fig. 10).

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